corresponds to H in a similitude of ratio $-1/3$, the center of similitude being N.[1] But N is also the nine-point center of the triangle BCH, whose orthocenter is A, hence the centroid G_1 of BCH corresponds to A in a similitude of ratio $-1/3$ with N as center of similitude. Similarly for the centroids G_2, G_3, of the triangles CHA, HAB. Consequently: *The four centroids of an orthocentric group of triangles form an orthocentric group, the two groups being similar and similarly placed.*

8. Since the centroids G, G_1, G_2, G_3, form an orthocentric group, all the properties of such a group immediately follow, as, for instance, that G is the orthocenter of the triangle $G_1G_2G_3$, etc.

Again the similitude of the two groups $GG_1G_2G_3$ and $HABC$ puts into evidence a great many properties, as for instance, that G_1G_2 is parallel to AB and is equal to 1/3 of its length; that the point of intersection of GG_1 and G_2G_3, which will be represented by (GG_1, G_2G_3), is collinear with N and $D \equiv (HA, BC)$; etc. The reader may find it interesting to formulate a number of these propositions.

9. In the similitude (7) by which the group $GG_1G_2G_3$ is derived from the group $HABC$, the center of similitude N is a double point. Hence: *An orthocentric group of triangles and the orthocentric group of their centroids have the same nine-point center.*

10. The orthocentric group $GG_1G_2G_3$ has been derived from the given orthocentric group $HABC$ by a similitude of center N and ratio $-1/3$. But the process may be reversed, and the orthocentric group $HABC$ may be derived from the orthocentric group $GG_1G_2G_3$, considered as given, by a similitude of ratio -3, the center remaining the same. Consequently: *The four points of an orthocentric group may be considered as the centroids of another orthocentric group of triangles, the two groups having the same nine-point center, this point being a center of similitude of the two groups, the ratio of similitude being* -3.

11. Since from (1) the two groups $HABC$ and $OO_1O_2O_3$ are symmetrical about the center N, therefore it follows from (10) that the two groups $GG_1G_2G_3$ and $OO_1O_2O_3$ admit N as a center of similitude, the ratio of similitude being $+3$. Hence: *The centroids and the circumcenters of an orthocentric group of triangles form two orthocentric groups of points having the same nine-point center, this point being a center of similitude of these two groups, the ratio of similitude being* $+3$.

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C. Maclaurin's *Geometria organica sive descriptio linearum curvarum universalis*, published at London—G. Poleni's *De mathesis in rebus physicis utilitate praelectio habita* · · ·, published at Patavia—Second edition of L'Hospital's *Traité analytiques des sections coniques*, published at Paris—Alexandre Savérien, author of *Dictionnaire universel de mathématiques et de physique* (2 vols., Paris, 1753), born July 16.

[1] Euler, *Novi comment. acad. sc. Petrop.*, vol. 11 (1765), 1767, p. 114.—EDITOR.